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Are No-Ponzi Game and Transversality Conditions Relevant for Public Debt? A Keynesian Appraisal

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Abstract:
This paper investigates the relevance of the No-Ponzi game condition for public debt (i.e. the public debt growth rate has to be lower than the real interest rate) and the transversality condition for the GDP growth rate (i.e. the GDP growth rate has to be lower than the real interest rate). First, it appears on OECD data, that over the last 40 years, those conditions were validated only for 24% of the cases under examination. Second, the No-Ponzi and the transversality conditions were more frequent in the 1980s and the 1990s following changes towards more restrictive monetary policies. Third, in tune with the Keynesian view, the data show that cases where the real interest rate is lower than the GDP growth rate may also lead to public debt consolidation (i.e. a decrease in the debt to GDP ratio) in 26% of the cases, compared with only 19% corresponding to the textbook case in which both GDP and public debt growth rates are below the interest rate.

Keywords: Public debt solvency, No-Ponzi Game condition, transversality condition, Keynesian countercyclical policy, monetary policy, economic growth.
JEL Classification: E43, E5, E6, H6

Résumé :

Le présent article porte sur la pertinence, à partir des données de l'OCDE, de la condition d'absence de jeu de Ponzi pour la dette publique (c'est-à-dire que le taux de croissance de la dette doit être inférieur au taux d'intérêt réel) et de la condition de transversalité sur le taux de croissance du PIB (c'est-à-dire que le taux de croissance du PIB doit être inférieur au taux d'intérêt réel). Tout d'abord, il apparaît que sur les 40 dernières années, ces conditions n'ont été validées que dans 24 % des cas. Ensuite, les conditions d'absence de jeu de Ponzi et de transversalité ont été plus fréquentes durant les années 1980 et 1990, à la suite des changements vers des politiques monétaires plus restrictives. Troisièmement, conformément au point de vue keynésien, les données montrent que les cas où le taux d'intérêt réel est inférieur au taux de croissance du PIB correspondent elles aussi à des phases de consolidation de la dette (c'est-à-dire à une réduction du ratio de dette publique rapportée au PIB) dans 26 % des cas, en comparaison seuls 19 % des cas correspondent au cas d'école retenu dans les manuels de référence où les taux de croissance du PIB et de la dette publique sont censés être inférieurs au taux d'intérêt.

"The average realized real rate of return on government debt for major OECD countries over the last 30 years has been smaller than the growth rate. Does this imply that governments can play a Ponzi debt game, rolling over their debt without ever increasing taxes?" Blanchard and Weil (1992).

1. Introduction

This paper investigates the relevance of the No-Ponzi game condition for public debt[1] and the transversality condition[2] for the GDP growth rate, which have been endemic in graduate macroeconomic textbooks for the last 20 years. Their relevance is assessed first with respect to their ability to describe observed macroeconomic data, and second with respect to a normative macroeconomic policy view which concerns the long term solvency of public debt.

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1 The No-Ponzi game condition for public debt states that the public debt growth rate has to be lower than the real interest rate.

2 The transversality condition states that the growth rate of GDP (as well as capital) has to be lower than the real interest rate.

First, it appears in OECD data that over the last 40 years, those conditions were validated in only 24% of the cases under examination. Those conditions also depict a world where debt consolidation (i.e. the debt to GDP ratio decreases) occurs in around 80% of the above cases (i.e. 19% over a marginal total of 24% of the cases). With the overall data suggesting that consolidation occurred in 45% of the cases, the conditions above correspond to only 42% of the total consolidation occurrences. As a consequence, those textbooks considered in many economics departments as reference books bias the mind-sets of graduate students (some of them becoming future policy makers or economic advisors) with respect to what really happens in the economy.

Second, the empirical relevance of those conditions varied over the last four decades, depending on monetary and fiscal policy changes. The No-Ponzi and the transversality conditions were indeed more frequent in the 1980s and the 1990s following changes towards more restrictive monetary policies which led to a decrease in inflation. Third, after descriptive relevance (i.e. the concordance with the observed macroeconomic data), comes the normative point of view. The No-Ponzi Game condition is also considered as a normative policy which reflects the very long term solvency of public debt. However, the data show that cases where the real interest rate is below the growth rate of output may also lead to public debt consolidation (i.e. a decrease in the debt to GDP ratio) in 26% of all cases (58% of consolidation occurrences), which is more coherent with the Keynesian view according to which keeping the interest rate below the growth rate removes the "snowball" effect and re-establishes better control on public debt dynamics, with respect to the 19% (42% of consolidation) corresponding to the textbook case in which both GDP and public debt growth rates are below the interest rate.

When the transversality condition on the growth rate of output is not met (i.e. the growth rate of output is above the real interest rate), the increase in the debt to GDP ratio occurs in only 14% of all cases (25% of the observed debt to GDP ratio increases), whereas when the growth rate of output is below the real interest rate ("snowball" effect), the debt to GDP ratio increases in 41% of the cases (75% of the observed debt to GDP ratio increases). Here again, this is more consistent with the Keynesian framework of analysis.

From a Keynesian point of view, the No-Ponzi game condition is a self-contradictory norm for monetary policy because the debt growth rate depends positively on the real interest rate by virtue of straightforward accounting principles. Moreover, the transversality condition creates a "snowball" effect which increases the probability of the debt growth rate exceeding the interest rate, i.e. for the No-Ponzi game condition not to be met. Hence, those two principles are not likely to be dynamically compatible, and to try to satisfy both of them simultaneously is likely to create macroeconomic instability.

The paper is organized as follows. Section 2 discusses a few theoretical consequences of the No-Ponzi game and the transversality conditions in current graduate macroeconomic textbooks. The counter-cyclical Keynesian policy point of view with respect to those two conditions is then outlined in section 3. In section 4, a statistical analysis describes the occurrences of the above cases for OECD countries over the last 40 years with a split by decades. A short conclusion follows.

2. The No-Ponzi Game and the Transversality Conditions in Graduate Modern Macroeconomic Theory Textbooks

The No-Ponzi Game condition (henceforth the NPG condition) on public and private debt (which also stands for a transversality condition for debt) eliminates the possibility of a Ponzi chain letter by stating that the growth of private debt and of public debt has to be lower than the real interest rate charged on this debt in the very long run. More precisely, let us quote a textbook by two by Heijdra and Van Der Ploeg (2002, p.479):

"Provided that the agent has free access to the capital market, the choice of the problem so far is not meaningful: the agent can simply borrow an infinite amount, service the debt with further borrowings, and live in a state of utmost bliss (presumably that would mean "all fun and no work", with consumption tending to infinity and worked hours to zero). Obviously, something is missing in the story up to now to make for interesting macroeconomics. ***The key to the puzzle*** *is obtained by integrating the dynamic budget equation (the wealth accumulation or the flow of funds equation)."*

$$\dot{a} = (r-n).a_t - (C_t + T_t - W_t) \Rightarrow a_\tau = \int_\tau^{+\infty} (C_t + T_t - W_t) e^{-(r-n)t} dt + \left[\lim_{t \to +\infty} a_t e^{-(r-n)t} \right]$$

where the time index is denoted *t*, *n* is the exogenous growth rate of population, *a* is real financial assets per capita (when negative, it represents the household's debt), *r* is a real interest rate, *w* is the real wage, *T* is lump-sum tax per capita and *c* is consumption per capita of a homogenous good, according to the version of the Ramsey model.

Barro and Sala-I-Martin (2004), p.89 and p.92 make a distinction calling the transversality condition as the equality of the term in square bracket to zero: "*It would be suboptimal for households to accumulate positive assets forever at the rate r or higher, because utility would increase if these assets were instead consumed in finite time".* They define the no Ponzi game as an inequality condition such that the term in square brackets is at least positive. With their definition, the transversality condition imply the no Ponzi game condition. Their explanation is as follows: "*In order to borrow on this perpetual basis (schemes in which households debt grows forever at the*

rate r or higher), households would have to find willing lenders; that is, other households that were willing to hold positive assets that grew at the rate r or higher. But we already know from the transversality condition that these other households will be unwilling to absorb assets asymptotically at such a high rate".

When the growth rate of assets is lower than the real interest rate (i.e. when the NPG condition holds), the household intertemporal budget constraint says that the value of financial assets that the agent possesses in a given period must equal the present discounted value of the excess of consumption over after-tax labor income.

The same reasoning applies when adding governments in the Ramsey model (Heijdra and Van Der Ploeg, 2002, p.442). The government identity (in per capita form) is given by a differential equation that could be integrated:

$$\frac{\partial D}{\partial t} = \left(r(t) - n\right).D_t - \left(T_t - G_t\right) \Rightarrow D_\tau = \int_\tau^{+\infty} \left(T_t - G_t\right) e^{-(r-n)\cdot t} dt + \left[\lim_{t \to +\infty} D_t e^{-(r-n)\cdot t}\right]$$

Public debt per capita is denoted *D*, lump-sum taxes per capita are denoted *T*, government consumption per capita is denoted *G*, *r* is the interest rate on public debt, t is a time index. The No Ponzi game condition for government is such that:

$$\lim_{t \to +\infty} D_t e^{-(r-n)\cdot t} = 0 = \lim_{t \to +\infty} D_0 e^{(g_D - n)\cdot t} e^{-(r-n)\cdot t} \Rightarrow g_D < r$$

In the right hand side of the second equality, the growth rate of debt (denoted g_D) is assumed to be constant. In this case, the no Ponzi game condition is such that growth rate of debt has to be lower than the real interest rate. Public debt is then exactly equal to the present value of all the discounted future primary (positive or negative) surpluses. *With the NPG condition, it is necessary for the existence of a (positive) public debt that the value of the cumulated discounted future public deficits is more than offset by the value of the cumulated discounted future public surpluses.* There are infinitely many paths for future taxes and futures government expenditures.

The NPG condition is one of the three necessary assumptions in order to obtain "Ricardian equivalence" in Ramsey models with discounting. Ricardian equivalence states that budgetary policy has no effect on consumption and the Keynesian multiplier is equal to zero. More precisely, three assumptions are required for Ricardian equivalence (Barro (1974)): (1) Households maximize intertemporally an utility function with aversion with respect to consumption fluctuations without credit constraints, (2) Government finances only government consumption with lump-sum taxes and public debt (no public productive investment, no distortionary taxation, nor money creation), (3) the assets (or debt) owned by households and by government are exactly *zero* at the final date in finite horizon or infinite horizon. The transversality conditions are assumptions (3) for the infinite horizon

case. Once (1) and (2) are assumed, there is a mathematical equivalence: Ricardian equivalence holds if and only if the transversality conditions hold.

The transversality condition on the growth of accumulated capital are introduced the neo-classical model of investment, where this time, it is the growth of capital which has to be lower than the real interest rate used as a discount rate in the infinite horizon. It appears also in the endogenous growth literature for the AK model, where the endogenous growth of output, capital and consumption is limited by a ceiling derived from transversality condition that the balanced growth rate of capital, of output and of consumption has to be lower than the discount rate. The claim appears here that "utility has to be bounded".

However, the transversality conditions are built on shaky mathematical and economic grounds for the following reasons.

(1) It is the choice of a terminal condition for infinite horizon problems which is necessary only when lifetime utility is finite at the optimum (Kamihigashi, 2005). By analogy to a finite horizon terminal condition, it states that the discounted value of capital in the infinite horizon is zero. The Halkin (1974) counter-example demonstrates that *in general*, there are no necessary transversality conditions for infinite horizon optimal control problems when one does not assume that the objective function converges. Moreover, even when the objective function does converge in Halkin's (1974) counterexample, Caputo (2005, chapter 14) still concludes it is a valid counterexample for demonstrating that the usual textbook transversality condition is not necessary, contrary to the claim of Chiang (1992, Chapter 9). The first model of this type was proposed by Ramsey (1928) with an objective function without discounting. Ramsey did not assume those transversality conditions, and his model is still not considered as flawed.

(2) The households maximising utility should target infinite utility and not bounded utility, even if they die (in the infinity limit) with non-zero positive wealth, in particular in endogenous growth (see an example in Amable, Chatelain, Ralf (2010)). Else, an explicit utility function should describe the losses related to personal the taste against "wasted assets" with infinite negative value. In the Ramsey (1928) model, the utility is unbounded.

(3) *Something else is missing*. In particular, the agent (households, firms, government) may not have ***free unlimited access to the capital market***, which is another key to the puzzle than "*the*" key put forward in Heijdra and Van der Ploeg quote. Let us propose ***alternative solvency "collateral" constraints*** set by imperfect capital market instead of "the transversality conditions" add-on for perfect capital markets with "free" access. Setting credit constraints and covenants for repeated short run solvency at each future period is a more natural way to fight against Ponzi behaviour than an infinite horizon solvency constraint such as the No Ponzi Game

condition. Imagine that we apply a similar reasoning as in the financial accelerator: public debt is solvent based on the capacity to repay bonds based on next period expected distortionary taxes on output net of public expenditures. And let us assume the lenders expect it to hold for all future finite dates *t*:

$$(1+\mathrm{r}_t)\cdot B_t < \tau_{t+1}\cdot Y_t\left(1+g_{t+1}(Y)\right)-\mathrm{G}_t\left(1+g_{t+1}(G)\right).$$

If the expected growth in output is large and if the interest rate on public bonds is low (*which is just the opposite of the infinite horizon transversality constraint on output or capital*), this solvency constraint is likely to be respected. This is what bond holders may think about in the short run. They would like taxes to increase and public expenditures to fall, but would enjoy growth in output even more, as it increases the taxable base.

Imagine that this is the case for *all* future periods. Then public debt is always short-term solvent. Imagine that at the same time, the growth rate of output is equal to the growth rate of public debt, but is larger than the real interest rate on public debt. Then, the infinite horizon (i.e. The NPG) solvency constraint is not fulfilled, whereas the short run solvency constraint is always fulfilled. In this case, the infinite horizon solvency constraint is meaningless. In this context, we do not know whether Ricardian equivalence holds or not.

If one introduces uncertainty in the above setting, then a key issue for solvency may be related to the investors time horizon for the expected growth rate of the country. If they take into account the expected growth rate for the next ten years, solvency problems are very likely to be minimal, even when adding uncertainty related to the growth of output. But, if they take into account only the short run (next year's growth rate), they may over-lend and suddenly stop a few years later, quickly leaving this country's sovereign bonds market. In this case, an additional simultaneous equation is required, where the risk premium determining the interest rate of public debt depends on the probability of default which is related to the above equation.

Finally, these solvency and collateral capital constraints make for much more interesting macroeconomics than the infinite horizon solvency of the No-Ponzi game condition in macroeconomics for at least three reasons:

1. The Keynesian multiplier may be effective, which is not the case with Ricardian equivalence.
2. The No-Ponzi Game condition related to private agents rules bubbles of private assets out of the model. This assumption is used to rule out *the existence* of bubbles *prior to the infinite horizon* in macroeconomic models. This is consistent with the efficient financial market

hypothesis. But this is an issue in dealing with financial crises and the link between monetary and macro-prudential policy (Chatelain and Ralf, 2012).

3. In the endogenous growth literature, the transversality conditions are inconsistent with growth miracles. For growth miracles, the growth of output consistently exceeds the real rate of interest for several decades as was the case for Japan between 1960 and 1990 or China between 1990 and 2010 (Amable, Chatelain, Ralf, 2010).

Although the NPG condition is stated for the "long run" infinite horizon, several macroeconomic models consider a fixed value of the interest rate, so that this NPG condition holds for all periods, and it is not only a limit condition. In real data, any time can be the short run "now" and the long run of many years ago. Hence, we need to investigate short run properties of the inequalities related to the NPG condition and the transversality condition on output in pre-NPG conditions, namely, in non-modern macroeconomics such as IS-LM type Keynesian macroeconomics.

3. Countercyclical Keynesian policies with public debt and demand-led output

In line with early Keynesian authors such as Hansen and Greer (1942), Lerner (1943) and Domar (1944), public debt (defined as the sum of accumulated deficits) is first understood on the ground of the fiscal multiplier. If the economy does not use its full production capacity, then any increase in public spending will induce faster growth, since production is supposed to be demand-led. Tax cuts are said to induce the same type of adjustments but with less intensity and their impact on public debt dynamics can be somewhat different, but we do not examine this issue further in the present paper (Pucci and Tinel, 2011).

It is generally acknowledged that such demand increases which occur through public outlays should give rise to the highest possible multiplier effect if financed by debt rather than taxes. From this point of view, public debt does not really compete with the supply of private assets because savings are endogenous with respect to public spending. A saving level is induced by public spending through national income adjustments. For this reason, long term interest rates are not supposed to rise mechanically with public debt. Besides, public bonds and private assets are not competing against each other for funds because the requirements of portfolio diversification make them much more complementary than substitutable, as they bear different yields and risks.

As long as the system is not at full-employment, crowding out effects should be negligible (Arestis and Sawyer, 2004a, 2004b). The further the economy is from full-employment, the less

price adjustments play a role as opposed to quantity adjustments. As the money supply is endogenous, the government should at least control short term interest rates. A Keynesian monetary policy consists in keeping interest rates low enough as economic activity slows down to prevent the cost of private investment from being too high and too much of a deterrent to investment and also to keep the cost of public debt as negligible as possible. Raising real interest rates above the GDP growth rate for a prolonged period when the economy is not at full-employment is clearly not a good monetary policy prescription from a Keynesian point of view.

At this point, it is worth noting an important methodological difference between this framework and the NPG condition approach which presumes that this normative rule should apply whatever the situation. In contrast to the NPG condition approach, the Keynesian view cannot decree a policy rule without any reference to the economic context. In particular, it has to take into account the position of the economy in the business cycle and to assess the level of capacity utilisation. As the instability of the system is acknowledged, monetary and fiscal policy prescriptions are liable to vary considerably according to the macro situation. Nevertheless, the fact that most of the time --during the last several decades-- capitalist economies do not evolve at full employment leads to us emphasise some normative rules --like public spending deficit, and expansionary monetary policies-- more than others which should be followed near full capacity utilisation. Moreover, if ever the NPG and the transversality conditions were followed by a government then the Keynesian view contends that it would be likely not to lead to the result claimed by its proponents.

The transversality condition stipulates that the GDP growth rate has to be lower than the real interest rate: this is not difficult to obtain, but then public debt is likely to increase as a macroeconomic compensation because of a “snowball” effect. More public debt offsets less growth and then the first condition is less and less likely to be met. In other words the NPG and transversality conditions might be dynamically incompatible, as the two conditions are more or less conflicting it seems difficult to hold both of them for a long period of time. This doesn't mean that it cannot happen sometimes (probably most of the time just before big crashes). Let's go back now to the global Keynesian analysis of public spending and debt.

Once growth has been stimulated through public investment and/or final consumption, the resulting increase in national income leads in turn to an increase in tax receipts. At the end of the process (in the long run) if the size of the multiplier is greater than 1 (which is often assumed under reasonable hypotheses but not always empirically verified) the rise in output is expected to be more important than the rise in public outlays ($g_Y > g_G$) and the rise in tax receipts $g_T > 0$ is supposed to compensate at least partially for the initial additional public spending which reduces both public

deficit and debt. Note that this result is likely not to be observable instantaneously or on a very short period of time because of time lag and multiplier time processes.

If the real interest rate is "not too high" (lower than the GDP growth rate) the ratio of public debt to GDP is supposed to be smaller at the end of the process than at its beginning. Though the level of public debt is higher, it is compensated for by an even higher level of GDP. In other words, the growth rate of the nominal public debt measured on the whole process is expected to be smaller than the growth rate of the domestic revenue during the same period of time: $g_D < g_Y$.

Of course, this result depends heavily on the elasticity of tax receipts to growth and has to be amended if the real interest rate r at which the government is able to issue bonds is greater than g_Y. In this situation, the "snowball" effect implies that the government has to run a primary surplus just to stabilise its debt to GDP ratio. In a macroeconomic context where the condition $r > g_Y$ holds, any deficit spending leads to $g_D > g_Y$.

The fiscal multiplier is supposed to be used voluntarily by government so as to regulate aggregate demand and hence employment fluctuations, in particular to prevent the activity from dropping too much when the private components of demand are declining. Those mechanisms can also be used the other way round: a government can run public surpluses in order to reduce demand and hence limit the GDP growth rate if the economy is already at full-employment. Such a policy reduces the debt to GDP ratio.

The normative rules attached to this framework of analysis can be summarised as follows. The government has to behave in a countercyclical way: deficit spending when the growth declines, which increases public debt in the short run, and running surpluses when the growth increases, which reduces public debt in the short run.

If the government behaves in accordance with the previous basic Keynesian rules, then the following macroeconomic set-up is likely to happen: if g_Y is low then $g_D > g_Y$, i.e. the debt to GDP ratio increases; and if g_Y is high then $g_D < g_Y$, i.e. the debt to GDP ratio decreases. Note that in this framework, many Keynesian economists would consider it preferable to give priority to employment even when r is "high", i.e. $r > g_Y$. In other words, as long as full-employment is not realised, $g_D > g_Y$ is expected even when $r > g_Y$. The "snow-ball" effect cannot be considered as a deterrent factor to deficit spending for a Keynesian government as long as full-employment is not attained.

Let's define g_Y as "high" when private demand is sufficient to induce a reduction in unemployment and, conversely, define g_Y as "low" when private demand is not sufficient to improve the level of employment. It is possible to some extent, to specify the behaviour of government according to the macroeconomic situation which is simply characterised by the level of

growth and the order of r, g_D and g_Y.

The table 1 below summarizes such a classification of countercyclical policies in the short run and also displays macroeconomic situations with non-Keynesian rules of economic governance.

Table 1: Specification of the policy mix behaviour according to booms or recessions

Regime	if...	g_Y is low (recession)	g_Y is high (boom)
1	$g_D > g_Y > r$	Debt/GDP increases Keynesian Budgetary Policy Expansionary Monetary Policy	Pro-cyclical budgetary policy (To be avoided)
2	$g_D > r > g_Y$	Debt/GDP increases Keynesian Budgetary Policy Restrictive Monetary Policy	Pro-cyclical budgetary policy
3	$r > g_D > g_Y$	Debt/GDP increases Keynesian Budgetary Policy More restrictive Monetary Policy	Pro-cyclical budgetary policy
4	$r > g_Y > g_D$	Pro-cyclical budgetary policy	Debt/GDP decreases Restrictive Budgetary Policy More Restrictive Monetary Policy
5	$g_Y > r > g_D$	Pro-cyclical budgetary policy	Debt/GDP decreases Restrictive Budgetary Policy Restrictive Monetary Policy
6	$g_Y > g_D > r$	Pro-cyclical budgetary policy	Debt/GDP decreases Restrictive Budgetary Policy Expansionary Monetary Policy

If r is high, the Keynesian framework does not clearly specify the policy that should be adopted by the government during the upper side of the business cycle: is it necessary to run surpluses or not in order to reduce the debt to GDP ratio?

This presentation of discretionary fiscal policies needs a few additional comments relating to automatic stabilisers. When growth accelerates, public spending automatically slows down because less urgent public spending is required to aid people in facing unemployment and poverty, meanwhile more taxes are levied on revenues and transactions simply because these are increasing. As a result, public deficit and public debt to GDP ratio are automatically reduced with more growth. When growth is slowing down, the opposite result is obtained: more public deficit and higher public debt to GDP ratio. A typical Keynesian idea is that even so called automatic stabilisers are not sufficient to improve economic activity suitably (i.e. to reach the level at which is starts to create jobs again); if no discretionary expansionist policy is undertaken, then the economy is likely to

remain locked much longer in a situation where $g_D > g_Y$. Though such a situation seems to have persisted over time in Europe, during the last 30 years, it appears that governments in fact resorted to countercyclical (Keynesian) discretionary fiscal policies (Amable and Azizi, 2011).

4. Confronting the No-Ponzi Game condition with OECD data

Tables 2 below presents breakdowns of the net and gross public debts. The real interest rate takes into account the 10 year government bond yield, net of the GDP deflator.

Table 2: Gross Domestic Product growth rate g_Y , net public debt growth rate g_D , real 10 years government bonds yields (544 OECD annual observations)

	Expansionary monetary policy: **16%**	« Intermediate » real interest rate: **60%**	Textbook theory "Restrictive monetary policy": No Ponzi Game Condition: **24%**
Debt/GDP decreases: $g_D < g_Y$ **45%**	$r < g_D < g_Y$ 12/544=**2%**	$g_D < r < g_Y$ 131/544=**24%**	$g_D < g_Y < r$ **102/544=19%**
Debt/GDP increases: $g_D > g_Y$ **55%**	$r < g_Y < g_D$ 74/544=**14%**	$g_Y < r < g_D$ 198/544=**36%**.	$g_Y < g_D < r$ **27/544=5%**

According to table 2, over the last 40 years, the two conditions (i.e. the NPG and the transversality conditions) were validated on OECD data in only 24% of the cases. They depict a world where debt consolidation (i.e. the debt to GDP ratio decreases) occurs in around 80% of the above cases (i.e. 19% over a marginal total of 24% of the cases), whereas the overall data suggest that consolidation occurred in 45% of the cases. The two conditions characterize thus only 42% of the set of consolidation occurrences. As a consequence, those "reference" textbooks based on both conditions distort the judgement of graduate students with respect to what really happens in the economy.

The data also show that cases in which the real interest rate is lower than the output growth rate may lead to public debt consolidation (i.e. a decrease of the debt to GDP ratio) in 26% of the cases, with respect to only 19% corresponding to the textbook case where both the GDP and the public debt growth rates are below the interest rate. In other words, 58% of the consolidation situations do not correspond to the NPG and the transversality conditions, which is in tune with the

previous Keynesian appraisal of the two conditions.

When the transversality condition is not met (i.e. when the GDP growth rate is higher than the real interest rate), the increase in the debt to GDP ratio occurs in only 14% of overall cases, whereas it increases in 41% of the cases when the output growth rate is lower than the real interest rate. In other words, the debt to GDP ratio is increasing, despite the satisfaction of the transversality condition, in 75% of the cases, which is coherent with the "snowball" effect mentioned above by the Keynesian view.

We now turn to gross public debt, which has lower extreme values than net public debt, although textbook theory refers to net public debt. However, the non-parametric breakdowns of table 2 are only changed by a few percentages points for the full sample, as seen on table 4.

Focusing on gross public debt for the whole period from 1970 to 2008, we observe as many cases in which $g_D < g_Y$ (48 %) as cases in which $g_D > g_Y$ (52 %). The data also demonstrates that r is higher than g_Y for 60 % of the observations. Within this subgroup ($r > g_Y$), two thirds of cases also correspond to a situation in which $g_D > g_Y$, this corresponds to the "snowball" effect expected by the Keynesian view. When both g_D and r are higher than g_Y, r exceeds g_D in 78 % of the cases, so we might consider that r is pretty "high". Three quarters of the observations for which $g_D > g_Y$ also correspond to a situation in which $r > g_Y$ and 58 % of the observations for which $g_D < g_Y$ correspond to a situation in which $r < g_Y$. In other words, a significant majority of macroeconomic configurations are more consistent with the Keynesian framework than with the NPG/transversality one. Note that when both g_D and r are lower than g_Y, r is then higher than g_D in 74 % of the cases, which can be interpreted as a not particularly "low" real interest rate.

Accordingly, we can conclude that: (1) A growth rate of output higher than the interest rate is an efficient protection against the accumulation of public debt burden. This configuration seems to be compatible with an "intermediate" real interest rate. (2) Countries experiencing cumulative indebtedness ($g_D > g_Y$) for the most part (76 %) also experience higher interest rates than GDP growth rates. Such a configuration is far more likely to occur for "high" real interest rates ($r > g_D$ for 78 % of the sample). Cells on the diagonal represent 62% of the sample.

Table 3 presents the average values of the three key variables of interest over the last four decades, which signals marked contrasts: According to tables 3 and 4, the empirical relevance of the NPG and the transversality conditions varied over time with respect to monetary and budgetary policy changes. Both conditions where more frequent in the 1980s and the 1990s following changes towards more restrictive monetary policy which led to a decrease in inflation. A glance at tables 3

and 4 suggests that there might be a positive correlation between the real interest rate and the growth rate of public debt.

Table 3: Average GDP growth rate, average gross public debt growth rate, average real 10 years government bonds yields (562 OECD annual observations)

1970s (90 obs.)	***r = 1.5 %*** $<$ g_D*= 1.6 %* $<$ g_Y*= 4.1 %* *r<g_Y corresponds to 87.8% of the cases and consolidation $g_D<g_Y$ to 68.9%*
1980s (81 obs.)	g_Y*= 4.1 %* $<$ g_D*= 5.5 %* $<$ ***r = 5.8 %*** *r<g_Y corresponds to 6.2% of the cases and consolidation $g_D<g_Y$ to 35.9%*
1990s (163 obs.)	g_Y*= 2.5 %* $<$ g_D*= 4.4 %* $<$ ***r = 4.7 %*** *r<g_Y corresponds to 12.9% of the cases and consolidation $g_D<g_Y$ to 41.1%*
2000s (207 obs.)	g_D*= 1.7 %* $<$ ***r= 2.0 %*** $<$ g_Y*= 2.3 %* *r<g_Y corresponds to 51.7% of the cases and consolidation $g_D<g_Y$ to 52.2%*

Table 4: GDP growth rate, gross public debt growth rate and real interest rate including median values for the 2000s.

		$r < g_Y$	$r > g_Y$
$g_D < g_Y$	1970s (90 obs.)	66.7 %	2.2 %
	1980s (81 obs.)	5.0 %	30.9 %
	1990s (163 obs.)	10.4 %	30.7 %
	2000s (207 obs.)	35.3 % **g_D= -2.5 %** < *r* = 1.5 % < g_Y= 3.4 %	16.9 % **g_D= -0.8 %** < g_Y= 1.4 % < *r* = 2.8 %
	Whole period (562 obs.)	**27.7 %**	**20.3 %**
$g_D > g_Y$	1970s (90 obs.)	21.1 %	10.0 %
	1980s (81 obs.)	1.2 %	62.9 %
	1990s (163 obs.)	2.5 %	56.4 %
	2000s (207 obs.)	16.4 % *r* = 1.7 % < g_Y= 2.8 % < **g_D= 6.5 %**	31.4 % g_Y= 0.3 % < *r* = 2.6 % < **g_D= 6.4 %**
	Whole period (562 obs.)	**12.5 %**	**39.5 %**

Very low real interest rates relative to GDP growth rates is the main characteristic of the 1970s, since for 88 % of the sample $r < g_Y$. Among this subgroup ($r < g_Y$) we also observe that:

- For three quarters of the observations, $g_Y > g_D$.

- Observations for which $g_Y < g_D$ not only mean a slightly weaker g_Y (3.4 % compared to 4.9 % for the group for which $g_Y > g_D$) but also a stronger g_D (6.4 % against 0.1 % !).

- Real interest rates do not differ significantly among the two subgroups (1.4 % against 1.2 %).

On the other hand, during this period, for 31 % of the sample (against 52 % for the whole period) g_D exceeds g_Y (in spite of a relatively "weak" interest rate in 68 % of these cases). In the 1970s, "high" real interest rates are not a good predictor of increasing indebtedness whereas decreasing indebtedness is strongly associated with a low *r*.

The situation is quite different for the 1980s and 1990s, characterized by a strong increase in real interest rates, especially in the 1980s (5.8 % against 1.4 % during the 1970s) combined with a sharp deceleration in GDP growth rates (2.5 % against 4.1 % in the 1970s): consequently *r* exceeds g_Y for around 90 % of the cases (94 % in the 1980s and 87 % in the 1990s). Such a situation is associated with "cumulative" public indebtedness in two-thirds of the cases for which $g_D > g_Y$. Actually g_D surges from 1.4 % in the 1970s to around 5.0 % during the two following decades. As in the general case, the configuration in which $g_D > r > g_Y$ occurs more frequently (75 %) than the situation for which $r > g_D > g_Y$, both in the 1980s and the 1990s.

If we focus on the "column" for which $r > g_Y$, it is important to notice that:

- During the 1980s, subgroups for which $g_D < g_Y$ and $g_D > g_Y$ are distinguished mainly by differentiated public debt growth rates (0.3 % against 9.0 %!) whereas other variables remain pretty similar (5.5 % against 5.9 % for *r* and 2.4 % against 2.7 % for g_Y).

- If the configuration is very similar during the 1990s for g_D (-0.4 % in the first subgroup, +7.5 % in the second) and *r* (the rate was the same one for the two groups at 5.1 %; nevertheless, some countries experienced a lower *r* combined with faster output growth, which eventually allowed them to "reestablish" $r < g_Y$), it is far more contrasted in the 1990s with respect to GDP growth rates, since the median for g_Y was 3.2 % in the first subgroup compared to 1.8 % only in the second.

Finally, the 2000s are very interesting to analyze since the picture is far more balanced with respect to both columns and lines: actually, for more than half of the sample (52 %), $g_D < g_Y$, and for another large proportion, $r < g_Y$. We also observe that the cells on the diagonal represent 67 % of the cases. More precisely:

- When $r > g_Y$, we notice $g_D > g_Y$ in two thirds (65 %) of the cases; $r > g_Y$ and $g_D > g_Y$ implies $g_D > r$ in 87 % of the cases (12 percentage points more than in the two previous decades). Conversely, when $g_D > g_Y$, we also observe $r > g_Y$ in two thirds (66 %) of the cases.

- For $r < g_Y$, we observe $g_D < g_Y$ in two thirds (68 %) of the cases. Conversely, for $g_D < g_Y$, we observe $r < g_Y$ in two thirds (66 %) of the cases.

Note that r < g_Y and g_D < g_Y induce g_D < r in 85 % of observations during the 2000s, which sensibly contrasts with the 1970s (where we observe a strong majority of 57 %) and this is a sign that public debt reduction has not only been led by lower interest rates during this period (on average, r decreased sharply from 5.5 % in the 1980s to 2.1 % in the 2000s but also, g_D decreased from 5.5 % to 4.2 % in spite of the slowdown of g_Y, from 2.4 % to 1.9 %).

Nevertheless, it seems that the real interest rate remains correlated with economic growth. In the 2000s, countries for which g_D < g_Y succeeded in decreasing their public indebtedness whatever the level of the real interest rate. Furthermore, GDP growth rates are very different in the four different configurations.

Finally, the cells on the diagonal representing the "usual" Keynesian configurations always represent more than 67 % of the sample whatever the decade considered.

5. Conclusion

An economic world where the No-Ponzi Game and the transversality conditions are always valid, as it may happen in contemporary reference macroeconomic textbooks on hundreds of pages, may not reflect what happened in the OECD countries over the period 1970-2008. Hence, the doubts expressed by Blanchard and Weil (1992) related to the no Ponzi game condition and the real world upon the period 1960-1990 are still valid twenty years later. However, their prevalence was much larger during the 80s and 90s.

But the claim that the NPG condition and the transversality condition insure solvency and debt/GDP consolidation is not validated by the data which are significantly more in line with the Keynesian framework.

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