A new measure of liquidity premium*

Perry Mehrling† Daniel H. Neilson‡

31 January 2008

Abstract

Financial forwards and futures allow banks to align mismatched cash inflows and outflows arising from the activities of their clients in the real economy. Even with these contracts, however, banks can achieve only imperfect offsetting of cashflows, leaving them bearing residual liquidity risk. Banks charge a price for bearing this risk which is not directly measureable. Owing to the different cash-flow characteristics of the two types of contract, this liquidity-risk premium is embodied in the price difference between futures and forwards in interest rates and in foreign exchange. The premium can be measured indirectly from market data, and aids in an understanding of two asset-pricing anomalies of monetary economics.

Contents

1 Introduction 2

2 Patterns of cash flows 6
   2.1 Interest-rate forwards and futures ........................................ 9
   2.2 Foreign-exchange forwards and futures .................................. 12

3 Data 13
   3.1 Interest and exchange rates ................................................ 13
   3.2 Interest-rate futures ........................................................ 14
   3.3 FX Futures ........................................................................ 15

4 Measurement 16
   4.1 Liquidity premium: interest rates ......................................... 16
   4.2 Liquidity premium: foreign exchange ................................... 17

5 EH and UIP interpretation 19

6 Time varying risk premia 23
   6.1 Verification of the failure of EH ............................................. 23
   6.2 EH regression with liquidity-risk premium .......................... 24
   6.3 Verification of the failure of UIP† ......................................... 25
   6.4 UIP regression with liquidity-risk premium ........................... 26
   6.5 Observations ...................................................................... 27

*The authors wish to thank Suresh Sundaresan and participants in the Columbia Business School Finance Free Lunch and Macro Lunch for their comments. Errors are our own.
†pgm10@columbia.edu. Department of Economics, Barnard College, Columbia University
‡dhn2102@columbia.edu. Ph.D. student, Department of Economics, Columbia University.
Conclusion

Notation

Principles of notation:

- All interest rates are lowercase letters; all prices (including exchange rates) are uppercase.
- Numbers inside parentheses are dates. As a consequence, interest rates have two dates inside, exchange rates have one.
- Variables correspond to quoted numbers when possible.
- Subscripts are avoided when possible.
- Exchange rates are dollars per unit foreign currency, which reflects the asymmetry in currency pairs.

\[ F_t(N, T) \] quoted interest-rate futures price at \( t \) on a notional deposit from \( N \) to \( T \)

\[ h_t(N, T) \] implied futures rate, i.e. \( 1 - \frac{F_t(N, T)}{100} \)

\[ l(t, T) \] quoted \( T-t \)-day spot LIBOR at time \( t \)

\[ d(N, T) \] de-annualization factor for a deposit from \( N \) to \( T \), i.e. \( \frac{T-N}{360} \)

\[ f_0(N, T) \] annualized implied forward rate at time 0, i.e. \( \left( \frac{1+l(0, T)d(0, T)}{1+l(0, N)d(0, N)} - 1 \right) \frac{1}{d(N, T)} \)

\[ S(t) \] spot exchange rate (dollars per unit foreign)

\[ G_0(N) \] implied forward exchange rate at time 0 for delivery at \( N \) i.e. \( S(0) \frac{1+l(0, N)d(0, N)}{1+l(0, N)d(0, N)} \)

\[ G_t(N) \] quoted FX futures price at time \( t \) maturing at \( N \)

\[ l^*(t, T) \] quoted \( T-t \)-day spot foreign-currency LIBOR at time \( t \)

1 Introduction

The economy is chock full of natural forward positions. An American company orders a Swiss machine for delivery in three months, payable in Swiss Francs. This order involves a natural forward exchange position, the ultimate value of which depends on the spot exchange rate between dollars and Swiss Francs three months from now. The same company plans to finance its purchase by issuing dollar-denominated 90-day commercial paper three months from now. This plan involves a natural forward interest-rate position, the ultimate value of which depends on the spot rate of interest three months from now. If over the course of the next three months the exchange value of the dollar falls or the spot rate of interest rises, the company will lose on its natural forward positions, possibly so much so that the planned purchase no longer seems a wise business decision.

To avoid such potential losses, the company would like to hedge its natural forward positions. Ideally, it would like to find a counterparty that has exactly the opposite forward positions, which is to say a company with natural forward positions that will lose value if the exchange value of the dollar rises or the spot rate of interest falls. The coincidence, however, is unlikely. The ideal
counterparty must be expecting to receive Swiss Francs and planning to invest those Francs in a 90-day dollar-denominated instrument, and it must also be planning to make those transactions on exactly the same date and at exactly the same scale. Even if such a company were to exist, and even were the inevitable counterparty risk to prove tolerable, the cost of finding the company and then arranging a one-off bilateral deal is likely prohibitive.

Historically, the banking system arose to solve problems like this. For a fee, a bank sells Swiss Francs forward to the American company and so eliminates the exchange-rate risk. For another fee, it arranges a loan commitment that locks in the interest rate the company will pay. Now the company’s natural forward positions have become the bank’s formal forward contracts, and the company’s problem of finding an ideal counterparty has become the bank’s problem, but with some important differences. First, the bank has numerous clients with diverse payment needs, and some of those payment needs may offset, which means that the very same forward positions may pose less risk to the bank than they do to the company. More important, by dividing the company’s natural forward positions into two separate forward contracts, the bank can search for separate counterparties for each, perhaps even multiple counterparties for each contract.

We conceptualize the bank’s search for offset as beginning in the forward market with its regular banking counterparties, all of whom are trying to hedge their own net forward positions. Because the counterparties are familiar, counterparty risk in this market is controlled merely by limiting exposure to any one counterparty. But, just as the bank’s own forward positions are unlikely to offset exactly, so too in interbank trading the bank will in general have to be prepared to enter into offsetting forward contracts that don’t match its needs exactly. If a counterparty bank offers Swiss Francs at an attractive price but for a delivery date one week later than the bank has agreed with its client, it might be best to take the offer and plan to fill the gap by borrowing the Francs spot for a week, so accepting a week’s worth of exchange-rate risk, which is a lot better than three months. If a counterparty offers dollars to the right maturity date at an attractive rate, but for an issue date that is three days late, it might be best to take the offer and plan to fill the gap by borrowing dollars for three days, again trading a large interest-rate risk exposure for a small one. In practice, forward trading organizes itself around a collection of standard dates, and it costs more to issue a forward

---

1 For concreteness the following discussion focuses on forward and futures contracts, but the reader will observe that the argument extends to exchange swaps and forward rate agreements, as well as more exotic instruments. Similarly, the activity of the stylized banking system that we are discussing does not map directly on to the activities of a specific category of financial institutions, but is rather meant to highlight a specific dimension of the monetary and financial system as a whole.
that deviates from these standard dates.

The banking system’s search for offset continues on the futures exchange where futures contracts are even more standardized than forwards, in terms of issue date, tenor, and also contract size. Unlike the forward market, futures counterparties are anonymous so margin requirements and mark-to-market accounting are used to ensure performance. (In effect, the futures exchange takes on the task of managing the line of credit available to each exchange participant.) Because of the standardization, hedging forwards with futures is inevitably imperfect, but the advantage is liquidity, since the greater standardization brings deeper market participation, including speculators who have no natural forward position to offset, but rather are willing to take whichever side of the contract offers an attractive risk–return profile. Ultimately any imbalance in the natural forward positions emerging from the real economy will be shifted to speculators on the futures exchange, as much as possible.

Taken as a whole, it is clear that the banking system works to pool offsetting risks to the extent possible, and then to move whatever risk is left over onto the balance sheet of the agents most willing to bear it. In our stylized model, part of the residual risk stays on the bank balance sheet because client needs do not match standard forward or futures dates. The rest is shifted onto the balance sheet of speculators who take up the slack in the futures market. The important point is that, the more complete the offset in the forward market, the better terms banks can offer to their clients. The less complete the offset, the more reliance on speculators, and the higher the cost that banks will pass on as compensation for residual risk bearing.

It is important to emphasize this point. In taking on the natural forward positions of their clients, the banking system inevitably winds up shouldering some residual risk. How much risk is partly a matter of technology and institutional structure—wider networks and more efficient communication may help to achieve more complete offset—but it is also partly a matter of the pattern of natural forward positions emerging from the real economy. If the time pattern of planned future payments across the economy matched exactly, there would be no residual risk and hence no compensation for bearing it. Residual risk comes from the fact that planned future payments do not match. Although bearing such risk involves exposure to exchange-rate and interest-rate risk, the price of that risk will depend crucially on the degree of mismatch in the pattern of natural forward positions. This residual risk is thus best understood as liquidity risk, and the price is best understood as a liquidity premium. Banks can be expected to charge for bearing that risk, and the price can be expected to
vary over time, depending on the degree of payment offset achieved as well as on anything else that affects the cost of bearing that risk.

Table 1 shows a stylized diagram of the origin of residual liquidity risk that has guided our research and that provides the analytical framework for the empirical exercise that follows. It shows how the risk involved in myriad natural forward positions gets passed on to the banking system where it is netted out and the residual moved around using standard forward and futures contracts, as well as the notation we use for the rates and prices associated with these contracts. Presumably the full premium for bearing liquidity risk is included in the price that the bank offers to its client for a non-standard forward. But the difference between the non-standard forward and the standard forward might also include some liquidity premium, to the extent that the cash flow patterns diverge and so leave the bank holding residual risk. For the same reason, we might expect there to be some liquidity premium in the difference between the standard forward and the standard futures contract, even when the dates are the same, because the latter but not the former is marked to market. We might even expect to find some liquidity premium in the difference between the market price of the standard futures contract and the theoretical ideal valuation, unobservable in the real world, of that contract that takes into account only the price risk of the underlying exchange rate or interest rate, and hence is an unbiased forecast of the future spot rate.

<table>
<thead>
<tr>
<th>Contract</th>
<th>Client</th>
<th>Bank</th>
<th>Banking System</th>
<th>Exchange</th>
<th>Economic Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>Natural Forward</td>
<td>Non-standard Forward</td>
<td>Standard Forward</td>
<td>Standard Futures</td>
<td>Ideal Futures</td>
</tr>
<tr>
<td>Exchange rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: The origin and price of residual liquidity risk.

This last point requires further elaboration. Modern financial theory traces risk premia to systemic risk exposure, but largely abstracts from liquidity risk as a type of systemic risk exposure (Black 1976; Cox et al. 1981). When this theory is taken to the data, it is quite clear that there exist time-varying risk premia in the pricing of both exchange-rate and interest-rate futures; the empirical failure of uncovered interest parity for exchange rates and of the expectations hypothesis of the term structure for interest rates are two of the most dependable findings in the literature.
The problem is that standard financial theory provides little help in understanding why these premia exist—the usual sources of systematic risk seem too small to explain it (Tuckman 2002, p. 354; Grinblatt and Jegadeesh 1996, p. 1512). One remaining possibility however is that these premia reflect a source of risk that is not included in the standard theory, namely liquidity risk. Large mismatches in the pattern of natural forward positions can be expected to put pressure on futures markets that will distort prices away from their theoretical ideal.

The rest of this paper is best understood as an attempt to measure the liquidity premium in asset prices by using the difference between standard forward and standard futures prices, in both exchange-rate and interest-rate contracts, as a proxy. The main reason for this choice is data availability. We can obtain high quality data for both (implied) forward and futures contracts, and line up the dates of each exactly. Standard theory attributes any difference between the prices of these two contracts to (1) reinvestment risk involved in the mark-to-market feature of futures and (2) differential counterparty risk involved in forwards relative to futures. Neither of these seem to amount to much quantitatively in the markets we study. We propose instead to interpret any difference as a liquidity premium.

We choose to focus our empirical study on the short term Eurodollar market (and Euroyen, and Euribor) for two reasons. First, these markets are the deepest and most liquid interbank borrowing markets; if we are able to measure a liquidity premium in them, we can be sure of finding one elsewhere. Second, these markets are in practice where the banks involved in the payments system are most likely to turn for funding or to place short-term funds.

2 Patterns of cash flows

The payments problem solved by the banking system is to align mismatched patterns of cash flows stemming from natural positions in the real economy. To study the premium charged for the bearing of liquidity risk in providing this service, we study forward and futures contracts in interest rates and foreign exchange. The design of each of these contracts creates a specific pattern of cash flows. The use of this limited set of standardized contracts allows banks to achieve more complete offset

\[\text{Burnside et al. (2006); Piazzesi and Swanson (2006).} \]
of payments, but it remains unlikely that the offset will be perfect.

Forward contracts are traded in over-the-counter markets and are standardized in tenor, but are available in any size and can begin on any day. Futures contracts are traded on an exchange and are standardized in size and maturity date. The purchase of any of these contracts is equivalent to the purchase of the particular pattern of cash flows determined by the contract design, the purchase price, and the uncertain subsequent path of interest or exchange rates. Previous papers that have studied price differences between futures and forwards have mainly been interested in testing the efficiency of asset pricing in these markets, which they take to be a matter of convergence between futures and forwards (e.g. Grinblatt and Jegadeesh 1996, Sundaresan 1991). Because of this motivation, these papers have not generally been clear about the precise pattern of cash flows in question. Our story is entirely about that time pattern, so we must be much clearer.
Table 2: Timing of cash flows for Eurodollar futures and forwards.

<table>
<thead>
<tr>
<th>instrument</th>
<th>$0 &lt; t &lt; N$</th>
<th>$N$</th>
<th>$T$</th>
<th>$\text{obsv}'s$</th>
<th>at</th>
</tr>
</thead>
<tbody>
<tr>
<td>implied forward</td>
<td>0</td>
<td>0</td>
<td>$-1$</td>
<td>$1 + f_0(N, T)d(N, T)$</td>
<td>S</td>
</tr>
<tr>
<td>traded futures</td>
<td>$(F_t(N, T) - F_{t-1}(N, T)) \frac{d(N, T)}{100}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Q</td>
</tr>
<tr>
<td>refinanced forward</td>
<td>0</td>
<td>0</td>
<td>$-1 + f_0(N, T)d(N, T)$</td>
<td>0</td>
<td>S</td>
</tr>
<tr>
<td>Eurodollor forward</td>
<td>0</td>
<td>0</td>
<td>$(h_0(N, T) - l(N, T))d(N, T)$</td>
<td>0</td>
<td>NE</td>
</tr>
<tr>
<td>price-based futures</td>
<td>$\frac{1}{1 + (1 - \frac{2i(N, T)}{100})d(N, T)} - \frac{1}{1 + (1 - \frac{2i(N, T)}{100})d(N, T)}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>NE</td>
</tr>
</tbody>
</table>

*S: synthetic; NE: non-existent; Q: quoted

Table 3: Timing of cash flows for foreign-exchange forwards and futures.

<table>
<thead>
<tr>
<th>instrument</th>
<th>$0 &lt; t &lt; N$</th>
<th>$N$</th>
<th>$T$</th>
<th>$\text{obsv}'s$</th>
<th>at</th>
</tr>
</thead>
<tbody>
<tr>
<td>implied forward</td>
<td>$0$</td>
<td>$0$</td>
<td>$-F_0(N)$</td>
<td>S</td>
<td>0</td>
</tr>
<tr>
<td>f.c.</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>Q</td>
</tr>
<tr>
<td>futures</td>
<td>$G_t(N) - G_{t-1}(N)$</td>
<td>0</td>
<td>$-G_{N-1}(N)$</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>f.c.</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>N</td>
</tr>
</tbody>
</table>

*S: synthetic; Q: quoted
2.1 Interest-rate forwards and futures

Table 4 summarizes four interest-rate instruments along two of the dimensions which distinguish them. The instruments, of which two are hypothetical, are discussed in detail below. The left column shows contracts which settle to yield, including the actual Eurodollar futures contract and the hypothetical Eurodollar forward. The right column shows contracts which settle to price, including the hypothetical price-based futures and the implied forward. The top row shows contracts which are marked to market, i.e. futures contracts, and the bottom row shows contracts that are not marked to market, i.e. forward contracts. The meaning of each of these terms in terms of patterns of cash flows is elaborated in this section.

<table>
<thead>
<tr>
<th>Marked to market</th>
<th>Settles to yield</th>
<th>Settles to price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not marked to market</td>
<td>Eurodollar forward</td>
<td>implied forward</td>
</tr>
</tbody>
</table>

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 4: Comparison of interest-rate contract features.</td>
</tr>
</tbody>
</table>

Table 2 outlines the cash flows that can be obtained by purchasing various interest-rate contracts. All are shown from the point of view of the long side of the trade (that is, the side which profits from a subsequent decline in interest rates) and with a loan of $1 in mind. Consistent with market convention, interest rates are expressed in annual terms, which means that an adjustment factor is required to compute actual interest payments when the maturity is different from one year. This de-annualization factor we denote $d(N, T)$.

The first row of Table 2 shows the forward loan implied by spot LIBOR, the instrument in the lower-right corner of Table 4. A suitable combination of long and short Eurodollar deposits allows the lender to fix at time 0 the rate on a loan from time $N$ to time $T$. This nets a cash flow of 0 at time 0, −1 at time $N$ and the return of the 1 plus interest at time $T$. The interest earned on this forward loan can be expressed in terms of the relevant spot rates, $viz.$

$$f_0(N, T) = \left( \frac{1 + l(0, T)d(0, T)}{1 + l(0, N)d(0, N)} - 1 \right) \frac{1}{d(N, T)}. \tag{1}$$

This formula follows from forward interest parity, a pure arbitrage condition with strong empirical support. We think of the other contracts as variations on this pure forward.

---

3Note that the implied forward in Table 2 entails a cash flow of $1 + f_0(N, T)d(N, T)$ at time $T$. This is the same
The second contract we consider is the standardized interest-rate futures contract, found in the upper-left corner of Table 4, typified by Eurodollar futures traded on the Merc and elsewhere (see Section 3.2 below for more on the institutional features of Eurodollar futures). A key difference between futures and forwards is the mark-to-market feature of the former. Market participants are required to hold margin accounts at the exchange, and at the close of each trading day, the value of each contract is determined and payments flow from the losers to the winners. This reduces counterparty risk both for the investors and for the exchange (which is legally the counterparty to each trade) by limiting the degree to which any participant can be out of the money. Because money can be withdrawn from variation-margin accounts when the balance is sufficient, and must be deposited into them when insufficient, this process has cash-flow implications for the holders of futures positions. These are quantified in the second row of Table 2. On each day until settlement, 0 < t < N, the long side receives a payment reflecting the change in futures price, \((F_t(N, T) - F_{t-1}(N, T)) \frac{d(N, T)}{100}\) (or makes such a payment if this quantity is negative, i.e. if the futures price has fallen). On the final day, time N, the contract settles to prevailing LIBOR and the final payment is made. In Table 2 we omit the cash flows due to investing or financing margin payments. These are small relative to the amounts we consider, for example financing $25 for 90 days at 5.25% costs $25 \times 0.0525 \times \frac{90}{360} = $0.33.

The futures contract is cash-settled, meaning that although the final settlement depends on the spot interest rate for a deposit maturing at time \(T = N + 90\), no money changes hands after time \(N\). This is in contrast to the forward contract, where money flows in one direction at time \(N\) and in the other at time \(T\). To create a synthetic Eurodollar investment that would address this incompatibility by moving all cash flows to time \(N\), consider the addition to the implied forward of a third leg, which borrows at time \(N\) an amount requiring the payment of exactly \((1 + f_0(N, T)d(N, T))\) at time \(T\). At time \(N\), the available rate on such a loan is \(T - N\)-day spot LIBOR, \(l(N, T)\), so the amount to as the expression given in Sundaresan [1991] p. 415, since

\[
1 + f_0(N, T)d(N, T) = \frac{1 + l(0, T)d(0, T)}{1 + l(0, N)d(0, N)}
\]

\[
= \frac{1 + l(0, T)d(0, T)}{1 + l(0, T)d(0, T)} \frac{b(0, N)}{b(0, T)} = b(0, N)
\]

where, in Sundaresan’s notation, \(b(0, N)\) denotes the time-0 price of a $1 Eurodollar deposit maturing at time \(N\).
be borrowed is
\[
\frac{1 + f_0(N,T)d(N,T)}{1 + l(N,T)d(N,T)}.
\] (2)

At time \( T \), the proceeds from the original long leg yield the amount needed to pay off this loan. We call the composite position a “refinanced” forward, and it is included as the third row of Table 2. Note that, while the cash flows of the implied forward are fully determined at time 0, refinancing the forward at time \( N \) makes cash flows dependent on \( l(N,T) \), which is unknown until \( N \). Since this position does not differ from the basic implied forward in either mark-to-market or settlement features, it occupies the same cell of Table 4. The refinanced forward is analogous to the forward contract in Treasury bills, which settles at time \( N \) with delivery of the T-bill itself. Even so it is not the forward version of the traded Eurodollar futures contract.

Beyond the cash-settlement distinction just described, two mismatches in cash-flows between the implied Eurodollar forward and the Eurodollar futures contract remain. First, the futures is marked to market, so that cash changes hands every day until time \( N \). This is the difference that is the focus of standard finance theory. Second, the futures settles to yield while the forward settles to price. The forward settles to price because the Eurodollar deposit pays add-on interest. The futures settles to yield because it was modeled after the T-bill futures contract (Stigum and Crescenzi, 2007; Chance, 2006). We can understand these mismatches by considering two hypothetical contracts which would address them in turn. These would occupy the lower-left and upper-right corners of Table 4.

The hypothetical security that Sundaresan (1991) calls a “Eurodollar forward” lies in the lower-left corner of Table 4. Like the refinanced forward, the Eurodollar forward yields all its cash flows at time \( N \). In addition to this, it settles to yield in a manner analogous to Eurodollar futures (and distinct from actual Eurodollar deposits). Using the terms of the foregoing discussion of patterns of cash flow, the Eurodollar forward yields the same net cash flows as a Eurodollar futures contract purchased at the same price, but without the mark-to-market feature of the futures contract on days \( 0 < t < N \).

To complete Table 4 an alternative futures contract could be imagined. Retaining the mark-to-market feature, this would settle to price (i.e. to \( \frac{100}{1 + l(N,T)d(N,T)} \)), rather than to yield (i.e. to \( 100 - l(N,T) \)), providing, on day \( N \), exactly the amount needed to buy a Eurodollar deposit from time \( N \) to time \( T \) paying $100 at time \( T \). This would have the result that, unlike the existing
Eurodollar futures contract, a one-basis-point move in the price would entail a variation-margin payment that varied with the price. This mismatch is discussed in detail by Chance (2006). The final row of Table 2 shows such an instrument, which we refer to as a “price-based futures” and whose price we denote $J_t(N, T)$.

Standard theory says that the yield on traded futures should be greater than the yield on the implied forward, for two reasons—the mark-to-market effect and convexity.

Reinvestment risk depends on the covariance between the futures price and the rate of interest. In the case of interest-rate futures, the covariance leads us to expect futures to be priced below forwards, which is to say the futures rate should be greater than the implied forward rate (Tuckman 2002, pp. 350–54). At short horizons, however, this effect is negligible (Tuckman 2002, p. 354; Grinblatt and Jegadeesh 1996, p. 1512), and we will assume that it is exactly zero. Because our theory suggests that the futures rate should be less than the forward rate, this assumption biases us against finding the effect in empirical data.

2.2 Foreign-exchange forwards and futures

Table 3 shows the pattern of cash flows of two contracts in the foreign-exchange markets. Consistent with market practice, we think of foreign currency as a speculative asset. As such, all exchange rates are expressed as prices of that asset, i.e. in the form 

\[ \frac{\text{dollars}}{\text{unit of foreign currency}}. \]

Thus, an appreciation of the asset is represented by an increase in the exchange rate.

Table 3 shows both the domestic- and foreign-currency flows of a forward and a futures contract in FX. The implied forward, shown in the first row, is achievable through use of the spot exchange market and a suitable foreign-currency Euro deposit. The forward exchange rate shown, $G_0(N)$, is the rate implied by covered interest parity (CIP), namely

\[ G_0(N) = S(0) \frac{1 + l(0, N)d(0, N)}{1 + l^*(0, N)d(0, N)}. \]  

(3)

That CIP holds is strongly supported empirically. Note that all parameters of the forward transaction are known at time 0.

The second row of Table 3 shows the cash flows entailed by the FX futures contract. The mark-to-market process works in a manner similar to that described for interest-rate futures above. In contrast to those contracts, however, “physical” delivery of the underlying commodity is possible
in foreign exchange and is the normal practice if the futures contract is held until maturity. At settlement, the long side receives a quantity of foreign currency corresponding to one unit of domestic currency. By summing all of the domestic-currency payments made over the life of the contract, it can be seen that the long side has paid $G_\theta(n)$ for this, i.e. the futures exchange rate prevailing when the contract was purchased. As with interest-rate futures above, in Table 3 we ignore cash flows due to the financing or reinvestment of variation-margin payments, since these are small relative to the other sums at issue. Note that all variation-margin payments are made in the domestic currency, consistent with the notion that the foreign currency functions as a speculative asset. Further discussion of FX futures is in Section 3.3.

3 Data

This section describes the data used in this study. All data were acquired via Bloomberg.

3.1 Interest and exchange rates

We used daily British Bankers’ Association fixings of spot US dollar LIBOR and yen LIBOR, and daily European Banking Federation fixing of EURIBOR for 1, 2, . . . , 12 months. The data for dollar LIBOR and yen LIBOR start in about 1990 (depending on maturity), EURIBOR data begin in 1999. We chose these interest rates for two main reasons: first, these markets are the deepest and most liquid interbank borrowing markets, and so if we are able to measure a liquidity premium in them, we can be sure of finding one elsewhere; second, these markets are the place where the banks involved in the payments system are in practice likely to turn for funding or to place excess short-term funds. Implied forward interest rates were computed from these spot rates using (1) above.

Spot exchange-rate data is available for the same dates. We chose to focus on two currency pairs: dollar/euro and dollar/yen. As with the choice of short-term interest rates, these pairs were chosen for the depth and liquidity of the associated markets. These are the most efficient of markets, and so any anomalies cannot be attributed simply to market inefficiency. Forward exchange rates were computed for various maturities using (3) above.
3.2 Interest-rate futures

Eurodollar (ED) futures trade on a number of exchanges, which are tightly linked by the possibility of arbitrage. The largest of these in terms of volume and open interest is the Chicago Mercantile Exchange (CME). CME ED contracts are listed for ten years on the March quarterly schedule, with the four nearest-month contracts available as fill-ins. Thus, at the beginning of January 2008, quarterly contracts were available to December 2017, and January 2008, February 2008, April 2008, and May 2008 are the four serial contracts. Each contract settles in cash to 100 minus the 3-month British Bankers’ Association LIBOR fixing for a deposit with face value $1,000,000 prevailing at maturity, that is,

\[ F_N(N, T) \equiv 100(1 - l(N, T)). \]  

(4)

Prior to settlement, the value \( F_t(N, T) \) is quoted and is thought of as the “futures price”, but note that it is not a price in the normal sense. \( F_t(N, T) \) is best thought of as an index value, where the change in value of a long contract purchased on date \( T_0 \) to date \( t \) is equal to

\[ \frac{F_t(N, T) - F_{t_0}(N, T)}{100} \times 1,000,000 \times d(N, T). \]  

(5)

The de-annualization factor \( d(N, T) \) is always \( \frac{90}{360} = 0.25 \) for ED futures. The choice of contract terms has the consequence that a rise of 0.01 in \( F_t(N, T) \) (i.e. a one-basis-point drop in the implied interest rate) represents an increase in value for the long side of the futures contract of

\[ \frac{0.01}{100} \times 1,000,000 \times 0.25 = 25. \]  

(6)

Cash settlement is necessary for ED futures, as Eurodollar deposits are not transferable. The particular choice of contract structure has certain complexities. First, the index value is constructed from the interest rate by subtracting it from 100 so that a rise in the index represents a gain for the longs, i.e. the “lenders”. This feature was useful to those at the Merc who were accustomed to trading commodity futures as it allows the index to be thought of as behaving like a price. Secondly, the index price used for the ED contract was designed with the T-bill futures contract in mind. While T-bills are priced as discount instruments, however, Eurodollar deposits pay interest as add-on securities. So while the index value for T-bill futures can be thought of as the time-\( N \) price of the underlying, this is not the case for Eurodollar futures. [Chance (2006)] is clearest on this mismatch.
CME offers two Euroyen contracts, one settled against yen LIBOR (ticker MY), the other against TIBOR (EY). Settlement is to 100 minus the LIBOR (resp. TIBOR) rate on a 3-month deposit with face value ¥100,000,000. Contracts are quarterly on a March cycle, which lines up with the quarterly ED contracts. Neither set of Euroyen contracts has, however, the fill-in monthly contracts which are offered for Eurodollar and EURIBOR futures (see below). Euroyen futures contracts are cash-settled.

EURIBOR (ER) futures are traded through LIFFE and settle to the European Banking Federation’s EURIBOR fixing. Specifically, settlement is to 100 minus the annual rate on a three-month deposit with face value €1,000,000. Contracts are on the March quarterly schedule, but as with Eurodollar futures, and as distinct from Euroyen, EURIBOR futures have four serial months to fill in. EURIBOR futures contracts are cash-settled.

In summary, Euroyen and EURIBOR futures are strictly comparable to the Eurodollar futures contracts. This comparability and the depth of the Eurocurrency interest-rate futures markets provide further rationale for the choice of Euro interest rates for this analysis.

3.3 FX Futures

FX futures, and in particular USD/EUR and USD/JPY futures, trade on the CME and settle to spot exchange rates at maturity. The Euro FX futures contract is for €125,000 while the JPY is for ¥12,500,000. The futures prices are quoted as exchange rates (unlike the index quote used for interest-rate futures). Dollars are in the numerator for both exchange rates, consistent with market convention for dollars against euros but not for yen against dollars, which is normally quoted as ¥/$. Thus a recent quote gave 1.4666 for the Euro FX futures contract and 0.009381 for the JPY contract. Both products are on a March quarterly schedule, and neither has serial month contracts.

FX futures are settled by physical delivery, which in this case means the transfer of the correct amount of dollars and foreign currency between the long’s and the short’s accounts. These transfers are overseen by the clearinghouse. In practice, physical delivery is rare and normal practice is for counterparties to close out their exposures prior to delivery by assuming offsetting contracts in the same currency pair.
4 Measurement

4.1 Liquidity premium: interest rates

To measure empirically the price of liquidity risk embodied in the difference between forward and futures prices in interest rates, we proceed as follows. The forward interest rate comparable to the futures yield index is the forward-interest parity rate $f_t(N, T)$, that is,

$$f_t(N, T) = \left(\frac{1 + l(t, T)d(t, T)}{1 + l(t, N)d(t, N)} - 1\right) \frac{1}{d(N, T)}. \quad (7)$$

This expresses the interest rate available at time $t$ for a deposit from time $N$ to time $T$, expressed in annual terms.

The futures price $F_t(N, T)$, as noted in Section 3.2 above, should be thought of as an index showing the change in the contract’s value (and thus of variation-margin payments), and not as a price per se. $F_t(N, T)$ can be converted to a futures yield index $h_t(N, T)$ by the linear transformation

$$h_t(N, T) = 1 - \frac{F_t(N, T)}{100}. \quad (8)$$

The yield index $h_t(N, T)$ is in units comparable to the interest rate, expressed in annual terms in line with market convention, on a $1$ million deposit from time $N$ to time $T$.

The price of liquidity risk in interest rates we define to be the difference between these two rates, namely

$$D^i_t(N, T) = \left(\frac{1 + l(t, T)d(t, T)}{1 + l(t, N)d(t, N)} - 1\right) \frac{1}{d(N, T)} - \left(1 - \frac{F_t(N, T)}{100}\right)$$

$$= f_t(N, T) - h_t(N, T). \quad (9)$$

The calculation of $D^i_t(N, T)$ depends on the measurement of $l(t, N)$ and $l(t, T)$. As mentioned in Section 3.1, LIBOR is available for $N - t, L - t \in \{30, 60, \ldots, 360\}$. The calculation also depends on the measurement of $F_t(N, T)$, where $T = N + 90$ by the design of the contract. As discussed in Section 3.2, futures contracts mature once per month. As a result of these constraints, we are able to compute a series of $D^i_t(N, T)$ only for $N - t \in \{30, 60, 90\}$, with corresponding $T = N + 90$. The results of these calculations are shown in Figure 1. The series are plotted at date $N$, that is,
observations in different series corresponding to the same futures maturity day line up vertically.

Figure 1: Liquidity-risk premium (interest rates) on left-hand scale. Federal funds target rate on right-hand scale. $D_{ir}(N, T), (N - t, T - N) \in \{(30, 120), (60, 150), (90, 180)\}$.

One of the strengths of our approach is in the care with which we constructed the series of $D_{ir}$ and the series $D_{fx}$ discussed below. The matching process just described involves careful tracking of futures expiration dates and of implied forward maturities. An alternative approach, taken by Grinblatt and Jegadeesh (1996), is to interpolate forward data between available maturities to find forwards to match a futures with any number of days remaining. We have not adopted this approach here, though it could be an avenue for further research.

4.2 Liquidity premium: foreign exchange

To measure the price of liquidity risk implied by the prices of foreign-exchange futures and forwards, we proceed as follows. As above, the forward exchange rate $G_t(N)$, expressed as a quantity of dollars per unit of foreign currency, implied by covered-interest parity is

$$G_t(N) = S(t) \frac{1 + l(t, N)d(t, N)}{1 + l^*(t, N)d(t, N)}.$$

For dates when no interpolation is necessary, our procedure is exactly the same as that of Grinblatt and Jegadeesh. In line with standard theory, they expect $h_t(N, T) > f_t(N, T)$ (futures rate > implied forward rate), so they report $h_t(N, T) - f_t(N, T)$ and document convergence to near zero as markets become more efficient. We however expect $f_t(N, T) > h_t(N, T)$ and so report $f_t(N, T) - h_t(N, T)$.
As for interest rates above, and following the conceptual argument made in Section 1, we define the liquidity-risk premium to be the difference between the forward exchange rate and the futures exchange rate. We compute this difference as a dollar payoff, the return in dollars to buying one unit of foreign currency at the futures exchange rate \( G_t(N) \) and selling one unit of foreign currency at the forward exchange rate \( G_t(T) \). The difference is therefore

\[
D_{t}^{fx}(N) = \frac{1 + l(t, N)}{1 + l^*(t, N)} S(t) - G_t(T) \\
= G_t(N) - G_t(N),
\]

following the convention used by \textit{Burnside et al. (2006)} for the difference between futures and forwards.

As with interest rates above, data for \( l(t, N) \) and \( l^*(t, N) \) is available every day for \( N - t \in \{30, 60, \ldots, 360\} \), while data for \( G_t(N) \) is available only for a limited set of fixed \( N \). Therefore our series includes dates \( t \) on which \( G_t(N) \) was available such that \( N - t \) was equal to 30, 60, or 90. Because of the quarterly schedule of foreign-exchange futures, these series are themselves quarterly. The results of this calculation for various values of \( N - t \) are shown in Figure 2 for euros, and in Figure 3 for yen.

![Figure 2: Liquidity risk premium (foreign exchange), euros. \( D_{t}^{fx}(t + \Delta N), \Delta N \in \{30, 60, 90\} \).](image-url)
5 EH and UIP interpretation

Two asset-pricing anomalies lie at the very center of monetary economics. The first one has to do with the relationship between interest rates over different time horizons. The expectations hypothesis of the term structure (EH), although eminently plausible on theoretical grounds, simply fails empirically. Long-term interest rates are on average greater than the subsequent sequence of short term rates can explain. Put another way, the forward interest rates implied by the term structure are upwardly biased forecasts of realized future spot interest rates. In formal terms, $f_0(N, T) > El(N, T)$, where $f_0(N, T)$ is the forward rate at time 0 for the period from time $N$ to $T$, $l(N, T)$ is the analogous spot rate that will be realized at time $N$, and $E$ denotes expectations at time 0.

The interest-rate anomaly means that you should be able to make money by borrowing short and lending long, or (which is essentially equivalent) by taking a long position in an interest-rate forward contract. There is risk in this “EH speculation”. If the spot interest rate rises above the forward rate, you’ll face a capital loss on your long position, or have to sell the proceeds of your

5See for example Blinder (2004, pp. 74, 82).
6See Piazzesi and Swanson (2006) for documentation and references.
7The asserted equivalence follows from forward interest parity (see equation (1)). Throughout this paper we assume that this relationship holds identically.
forward contract in the spot market for less than you contracted to pay. But the spot rate may also fall below the forward rate, giving you a capital gain and allowing you to sell your forward deliverable for more than you paid. One way to understand the EH asset-pricing anomaly is that the probability that the EH speculation makes money is greater than the probability that it loses money. There is risk, but on average the speculation is profitable.

No one really knows why this is so. Empirically there is clearly a risk premium in the term structure, and it also seems that that risk premium varies over time, but it is not so clear why this should be so. The banking perspective adopted in this paper suggests that forward rates get pushed around by an imbalance in the natural forward positions emerging from the natural economy, because that imbalance gets passed on to the banking system (broadly construed) which insists on being compensated for bearing it. If forward rates are pushed above expected spot rates, one reason might be that there are more people who want to lock in future borrowing rates than want to lock in future lending rates.

Table 5, panel (a) provides a stylized example of the balance-sheet relationships that would tend to push forward rates above expected spot. In the natural economy there are some people with natural short forward exposures who want to cover by acquiring a long forward hedge. That hedge is shown in the first row of the table as an on-balance-sheet transaction with $T$-month lending and $N$-month borrowing. There are other people with natural long forward exposures who want to cover by acquiring a short forward hedge, shown in the second row. These positions offset to some extent, and the banking system only has to bear the net, shown in the table as a net short hedge on the third row that forces the banking system to hold a net long forward position. Such an imbalance would mean that the short forward positions have to pay speculators a premium. The difference \( f_0(N, T) - El(N, T) \) is a time-varying risk premium, and the relevant risk is liquidity risk.

Unfortunately we do not observe \( El(N, T) \), so we cannot observe the risk premium directly. However, the banking perspective adopted in this paper suggests further that that premium can be divided into two pieces, the difference between the forwards \( f_0(N, T) \) and futures \( h_0(N, T) \), and the difference between futures \( h_0(N, T) \) and the expected spot rate \( El(N, T) \). The latter is still unobservable, but the former we can see in market prices. Under the hypothesis that the two pieces of the premium move together, we can use the former as a measure of the overall liquidity premium.

Thus the measurement exercise in Section 4 can be interpreted as measuring a time-varying liquidity premium. If this measure in fact captures the liquidity premium, it follows that it should
help us to forecast future spot rates. This will be the exercise in the next section, Section 6.

(a)

| Long forward | 50 | $T$-month dep. | $N$-month dep. |
| Short forward | 100 | $N$-month dep. | $T$-month dep. |
| Net short      | 50  | $N$-month dep. | $T$-month dep. |

(b)

| Long dollar   | 50  | \(l\) deposit | \(l^*\) deposit |
| Short dollar  | 100 | \(l^*\) deposit | \(l\) deposit |
| Net short     | 50  | \(l^*\) deposit | \(l\) deposit |

Table 5: The Possible Origin of the EH (a) and UIP (b) Anomalies

The second anomaly has to do with the relationship between interest rates in different currencies. Uncovered interest parity (UIP), another eminently plausible idea on theoretical grounds, fails even more resoundingly on empirical grounds. Currencies for which the nominal rate of interest is high not only fail to depreciate sufficiently against currencies for which the rate of interest is low, but they even tend to appreciate\(^8\). The difference between \textit{ex post} realized returns from investing in different currencies is thus even greater than the difference between \textit{ex ante} quoted returns. Put another way, the forward exchange rate implied by the pattern of interest rates is a biased forecast of the realized future spot exchange rate. In formal terms, when \(G(T) > S(0)\) (i.e. \(l^* < l\)) we also have \(G(T) > ES(T)\) where \(G\) is the forward exchange rate and \(S\) is the analogous spot exchange rate that will be realized at time \(T\) (both expressed in dollars per unit of the foreign currency).

The exchange rate anomaly means that you should be able to make money by borrowing in the low-interest-rate currency and lending in the high interest rate currency, or (which is essentially equivalent) by buying the high-interest-rate currency forward\(^9\). There is risk involved in this “UIP speculation”. The exchange value of the long currency position may fall by more than the interest differential, or the value of the forward contract may fall sufficiently to leave you with a net loss. But the exchange value may move the other way too, and the UIP anomaly consists in the empirical fact that it does so more often than not, so that the UIP speculation tends to make money on average.

---

\(^8\)See [Burnside et al., 2006] for documentation and references.

\(^9\)The asserted equivalence follows from covered interest parity (see equation (3)). Throughout this paper we assume that this relationship holds identically. Observe from this formula that \(l^* > l\) implies \(G(T) < S(0)\), and vice versa.
No one really knows why this is so either. Empirically there is clearly a risk premium, and it also seems that that risk premium varies over time, but it is not so clear why this should be so. The banking perspective adopted in this paper suggests that forward rates get pushed around by an imbalance in the natural forward positions emerging from the natural economy, because that imbalance gets passed on to the banking system (broadly construed) which insists on being compensated for bearing it. The UIP anomaly is that currencies with high rates of interest (equivalently, currencies that sell at a forward premium) tend to have their forward exchange rates pushed above expected spot exchange rates. The banking perspective suggests that this happens because there are more people who want to lock in the rate at which they can sell the high-interest-rate currency than there are people on the other side who want to lock in the rate at which they can buy the high-interest-rate currency.

Table 5, panel (b) provides a stylized example of the balance-sheet relationships that would tend to push the forward exchange rates of high-interest currencies above expected spot exchange rates. In the natural economy there are some people with natural short forward exposures who want to cover by acquiring a long forward hedge, shown in the first row of the table as an on-balance sheet transaction with borrowing in the foreign (low interest) currency and lending in the dollar (high interest) currency. There are other people with natural long forward exposures who want to cover by acquiring a short forward hedge, shown in the second row. These positions offset to some extent, so the banking system only has to bear the net, shown in the table as a net short hedge on the third row. Such an imbalance would mean that the short forward positions have to pay speculators a premium. The difference $G(T) - ES(T)$ is a time-varying risk premium, and the relevant risk is liquidity risk.

Unfortunately we do not observe $ES(T)$, so we cannot observe the risk premium directly. However, the banking perspective adopted in this paper suggests that that premium can be divided into two pieces, the difference between the forwards $G(T)$ and futures $G(T)$, and the difference between futures $G(T)$ and the expected spot rate $ES(T)$. The latter is still unobservable, but the former we can see in market prices. Under the hypothesis that these two pieces of the premium move together, we can use the former as a measure of the overall liquidity premium.
6 Time varying risk premia

This interpretation of the EH and UIP anomalies as arising from the fundamental liquidity problem of banking suggests a natural empirical test: does our measure of the price of liquidity risk, derived from the prices of instruments with different patterns of cash flows, help explain observed deviations from the EH and UIP null hypotheses? If so, these terms will be significant when added to the standard regressions used to test these hypotheses.

6.1 Verification of the failure of EH

The expectations hypothesis of interest rates states that forward interest rates should be equal in expectation to subsequent realized spot rates, i.e.

\[ f_0(N, T) = \mathbb{E}_0 l(N, T). \] (14)

This equality has been rejected in different forms by the literature (Fama and Bliss, 1987; Campbell and Shiller, 1991; Piazzesi and Swanson, 2006). Empirical tests are based on regressions of realized spot rates on past forward rates. Following Piazzesi and Swanson (2006), we tested the expectations hypothesis for spot and forward interest rates in our data. The simple version of the test we employed involves regression of the forecast error on a constant. The estimated constant will be equal to the mean of the forecast error, and so EH is rejected if the constant is different from zero. Table 6 reports the results of EH regressions of the form \[ 10000(f_0(N, T)d(N, T) - l(N, T)d(N, T)) = \alpha + \varepsilon, \] for various values of \( N \) and \( T \). \( f_0(N, T) \) is the forward interest rate for delivery at \( N \) and maturity at \( T \). \( l(N, T) \) is the spot interest rate available at (and observed at) \( N \) for maturity at \( T \). Values are de-annualized and interest rates are in basis points. Results vary with both \( N \) and \( T \), but in all cases we reject EH. The estimates for \( \alpha \) are consistent with those found in the literature.
Table 6: \( 10000(f_0(N,T) d(N,T) - l(N,T) d(N,T)) = \alpha + \varepsilon. \) Row headings are \( T - N \), column headings are \( N \). Under the EH null, \( \alpha = 0 \). Interest rates are in basis points.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 & 30 & 60 & 90 & 120 & 180 \\
\hline
 & (0.294) & (0.439) & (0.562) & (0.717) & (1.177) \\
 & (0.311) & (0.356) & (0.529) & (0.704) & (1.176) \\
 & (0.267) & (0.359) & (0.570) & (0.755) & (1.116) \\
120 & 17.380 & 15.833 & 15.863 & 17.955 & 22.663 \\
 & (0.287) & (0.421) & (0.553) & (0.761) & (1.187) \\
 & (0.347) & (0.408) & (0.575) & (0.781) & (1.064) \\
\hline
\end{array}
\]

6.2 EH regression with liquidity-risk premium

To test the significance of our measure of liquidity risk, derived from price differences between interest-rate futures and forwards, in explaining the observed deviation of interest rates from the EH null, we repeated the regression of Section 6.1 with the liquidity-risk premium \( D_{ir} \) as an additional term.

Because \( D_{ir} \) represents the forward rate less the futures rate, we expect its coefficient to be positive when we estimate the model. From the discussion in Section 5 if more people want to lock in forward borrowing rates than want to lock in forward lending rates (that is, if the real economy is net short forwards), the banking system is forced to hold a net long position and charges a premium for doing so. This premium, according to the view presented in Section 1 should be observable in \( D_{ir} \). Thus to the extent that implied forward rates are greater than futures rates, forward rates will deviate more from the EH null. Equivalently, we expect \( \beta > 0 \) in the regressions below.

The results appear in Table 7. This table should be compared with the 3rd row of Table 6 presented as Table 8 for easier comparison.
Table 7: 10000(\(f_0(N,T)d(N,T) - l(N,T)d(N,T)\)) = \(\alpha + \beta D_{ir} + \varepsilon\). Column headings are \(N\). \(T - N = 90\). Under the EH null, \(\alpha = 0\). Interest rates are in basis points.

<table>
<thead>
<tr>
<th></th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\text{1.506}))</td>
<td>((\text{2.115}))</td>
<td>((\text{3.422}))</td>
<td>((\text{5.315}))</td>
<td>((\text{8.376}))</td>
<td></td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.609</td>
<td>0.796</td>
<td>0.251</td>
<td>1.106</td>
<td>2.323</td>
</tr>
<tr>
<td>((\text{0.252}))</td>
<td>((\text{0.299}))</td>
<td>((\text{0.480}))</td>
<td>((\text{0.801}))</td>
<td>((\text{1.253}))</td>
<td></td>
</tr>
<tr>
<td>(N)</td>
<td>152</td>
<td>152</td>
<td>149</td>
<td>113</td>
<td>118</td>
</tr>
</tbody>
</table>

Table 8: EH regression without liquidity risk. Equivalent to 3rd row of Table 6. 10000(\(f_0(N,T)d(N,T) - l(N,T)d(N,T)\)) = \(\alpha + \varepsilon\). Column headings are \(N\). \(T - N = 90\) for comparison with Table 7. Under the EH null, \(\alpha = 0\). Interest rates are in basis points.

<table>
<thead>
<tr>
<th></th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>14.218</td>
<td>15.341</td>
<td>13.193</td>
<td>15.093</td>
<td>22.187</td>
</tr>
<tr>
<td>((\text{0.267}))</td>
<td>((\text{0.359}))</td>
<td>((\text{0.570}))</td>
<td>((\text{0.755}))</td>
<td>((\text{1.116}))</td>
<td></td>
</tr>
<tr>
<td>(N)</td>
<td>6101</td>
<td>6101</td>
<td>7500</td>
<td>4212</td>
<td>5519</td>
</tr>
</tbody>
</table>

Table 7 shows that \(\beta\), the coefficient on the liquidity premium \(D_{ir}\), is statistically significant and of the expected sign, indicating that \(D_{ir}\) helps to explain deviations of interest rates from the EH null. \(\beta\) is statistically significant at the 5% confidence level for \(N = 30\) and \(N = 60\), and at the 10% level for \(N = 180\). This is despite the paucity of observations in these regressions, which is due to our insistence on having a clean match between forwards and futures in constructing \(D_{ir}\).

A second effect is that, with the addition of \(D_{ir}\), \(\alpha\) decreases in all cases except \(T = 90\). This indicates that the addition of our measure of liquidity-risk premium reduces the average deviation of the observed rates from the EH null. We conclude that liquidity-risk premium aids in resolving this asset-pricing anomaly.

### 6.3 Verification of the failure of UIP

Uncovered interest parity is the hypothesis that forward exchange rates are an unbiased predictor of future spot exchange rates. Tests of UIP take the form of regressions of the realized appreciation or depreciation on the past forward premium or discount and a constant. UIP is rejected if the estimated constant is different from zero or the coefficient on the past forward premium or discount is different from one. Table 9 shows the results of UIP regressions using the specification of Burnside et al. (2006) on our data. \(S'(N)\) is the spot exchange rate available at time \(N\) in foreign currency dollars.
(we deviate from the convention used elsewhere in this paper to conform to the notation used by Burnside et al. (2006)). $G'_{N}(N + 1)$ is the forward exchange rate available at time $N$ for settlement at time $N + 1$. The unit of time is taken to be 30, 60, and 90 days. Consistent with the literature, we reject UIP. Moreover, as is usual, we find that currencies at a forward premium tend to appreciate and vice versa, exactly opposite to the claim of UIP.

$$S'_{N+1} - S'_{N} = \alpha + \beta G'_{N}(N + 1) - S'_{N} + \varepsilon.$$ Under the UIP null, $\alpha = 0$ and $\beta = 1$.

### Table 9

<table>
<thead>
<tr>
<th></th>
<th>JPY 30</th>
<th>JPY 60</th>
<th>JPY 90</th>
<th>EUR 30</th>
<th>EUR 60</th>
<th>EUR 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.006</td>
<td>-0.011</td>
<td>-0.019</td>
<td>-0.003</td>
<td>-0.007</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.007)</td>
<td>(0.010)</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.772</td>
<td>-1.533</td>
<td>-2.406</td>
<td>-1.652</td>
<td>-3.172</td>
<td>-5.070</td>
</tr>
<tr>
<td></td>
<td>(0.358)</td>
<td>(0.783)</td>
<td>(1.052)</td>
<td>(0.690)</td>
<td>(1.300)</td>
<td>(2.328)</td>
</tr>
<tr>
<td>$N$</td>
<td>209</td>
<td>104</td>
<td>69</td>
<td>97</td>
<td>48</td>
<td>32</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.022</td>
<td>0.036</td>
<td>0.072</td>
<td>0.057</td>
<td>0.115</td>
<td>0.137</td>
</tr>
</tbody>
</table>

6.4 UIP regression with liquidity-risk premium

To test the significance of our measure of liquidity risk, derived from price differences between FX futures and forward exchange rates implied by CIP, in explaining observed deviations from UIP, we repeated the regression of Section 6.3 with the liquidity-risk premium as an additional term.

Because $D^\text{fx}$ represents the forward price less the futures price, we expect its coefficient to be positive when we estimate the model. From the discussion in Section 5, if more people want to lock in the price at which they can sell the high interest-rate currency than want to lock in the price at which they can buy it, the banking system will be forced to hold a net long forward position in the high interest-rate currency and will charge a liquidity-risk premium for doing so. This premium should be observable in $D^\text{fx}$. To the extent that forward exchange rates are greater than futures rates, forward rates will deviate more from the UIP null. From the argument in Section 5, we expect $D^\text{fx} > 0$ to imply that $G > ES(N + 1)$. In the regressions we use the reciprocal exchange-rate convention, so we this is equivalent to $G'_{N} < ES'(N + 1)$. Thus we expect $\gamma > 0$ in the regressions.

The results appear in Table 10. Table 9 is reprinted as Table 11 for comparison.
\[ \frac{S'(N+1) - S'(N)}{S'(N)} = \alpha + \beta \frac{G_f(N+1) - S'(N)}{S'(N)} + \gamma D_{fx} + \varepsilon. \]

<table>
<thead>
<tr>
<th></th>
<th>JPY 30</th>
<th>JPY 60</th>
<th>JPY 90</th>
<th>EUR 30</th>
<th>EUR 60</th>
<th>EUR 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.004</td>
<td>-0.005</td>
<td>-0.014</td>
<td>-0.005</td>
<td>-0.006</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.009)</td>
<td>(0.012)</td>
<td>(0.006)</td>
<td>(0.008)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.216</td>
<td>-1.106</td>
<td>-2.071</td>
<td>-1.659</td>
<td>-4.010</td>
<td>-3.733</td>
</tr>
<tr>
<td></td>
<td>(0.782)</td>
<td>(1.008)</td>
<td>(1.252)</td>
<td>(1.283)</td>
<td>(1.940)</td>
<td>(2.383)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-2.532</td>
<td>0.717</td>
<td>-1.858</td>
<td>-8.231</td>
<td>-5.192</td>
<td>8.599</td>
</tr>
<tr>
<td></td>
<td>(3.992)</td>
<td>(2.100)</td>
<td>(5.184)</td>
<td>(5.275)</td>
<td>(6.038)</td>
<td>(7.758)</td>
</tr>
<tr>
<td>( N )</td>
<td>69</td>
<td>68</td>
<td>68</td>
<td>32</td>
<td>32</td>
<td>31</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.006</td>
<td>0.019</td>
<td>0.047</td>
<td>0.105</td>
<td>0.145</td>
<td>0.164</td>
</tr>
</tbody>
</table>

Table 10: \( \frac{S'(N+1) - S'(N)}{S'(N)} = \alpha + \beta \frac{G_f(N+1) - S'(N)}{S'(N)} + \gamma D_{fx} + \varepsilon. \)

\[ \frac{S'(N+1) - S'(N)}{S'(N)} = \alpha + \beta \frac{G_f(N+1) - S'(N)}{S'(N)} + \varepsilon. \]

<table>
<thead>
<tr>
<th></th>
<th>JPY 30</th>
<th>JPY 60</th>
<th>JPY 90</th>
<th>EUR 30</th>
<th>EUR 60</th>
<th>EUR 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>-0.006</td>
<td>-0.011</td>
<td>-0.019</td>
<td>-0.003</td>
<td>-0.007</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.007)</td>
<td>(0.010)</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>-0.772</td>
<td>-1.533</td>
<td>-2.406</td>
<td>-1.652</td>
<td>-3.172</td>
<td>-5.070</td>
</tr>
<tr>
<td></td>
<td>(0.358)</td>
<td>(0.783)</td>
<td>(1.052)</td>
<td>(0.690)</td>
<td>(1.300)</td>
<td>(2.328)</td>
</tr>
<tr>
<td>( N )</td>
<td>209</td>
<td>104</td>
<td>69</td>
<td>97</td>
<td>48</td>
<td>32</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.022</td>
<td>0.036</td>
<td>0.072</td>
<td>0.057</td>
<td>0.115</td>
<td>0.137</td>
</tr>
</tbody>
</table>

Table 11: \( \frac{S'(N+1) - S'(N)}{S'(N)} = \alpha + \beta \frac{G_f(N+1) - S'(N)}{S'(N)} + \varepsilon. \) Under the UIP null, \( \alpha = 0 \) and \( \beta = 1. \)

Note that \( \gamma \) is not significantly different from zero in any case. This is certainly due in part to paucity of observations, a consequence of the matching procedure between forwards and futures and the fact that foreign-exchange futures trade only quarterly. Note, however, that the estimates for \( \beta \), which should be 1 under the UIP null, move toward the theoretical value when \( D_{fx} \) is added as an explanatory variable. While this evidence cannot be considered conclusive, it does suggest that the liquidity-premium argument we presented in Sections 1 and 5 may offer part of an explanation for the anomaly.

### 6.5 Observations

Our desire to limit ourselves as far as possible to observable prices reflecting actual trading opportunities available to market participants creates some limitations for these regressions. In particular, and as discussed above, since futures contracts trade with fixed maturity dates, and since Eurocurrency deposits (and hence our implied forwards) trade every day but with fixed terms, the number of observations we are able to use is limited to the number of days on which the maturity of the futures
contract matches the maturity of the implied forwards. This occurs only once each quarter for each maturity for FX futures, which are listed quarterly, and only once each month for the interest-rate futures, which are listed monthly. For this reason, the EH regressions are limited to about 150 observations, while the UIP regressions are limited to about 70 observations for dollar–yen and about 30 for dollar–euro (owing to the longer series of yen FX futures).

One could increase the number of observations in these regressions by interpolation, for example by creating an entire term structure from the observed Eurodollar deposit rates. While this would allow comparison between futures and forwards on more days, we have avoided such measures because they add a layer of interpretation between our conclusions and the data, which largely reflects real trading opportunities available to market participants.

Our measure of liquidity risk helps to explain the failure of the expectations hypothesis of interest rates (EH). In explaining deviations of exchange rates from uncovered interest parity (UIP), though we see the average deviation decrease, the coefficient on the liquidity-risk premium itself is not statistically significant in our regressions. This fact may be due to the way in which banks internally manage credit risk with their foreign-exchange clients. If forward foreign-exchange transactions are internally marked to market, which would be a reasonable risk-management step to take in the highly volatile FX market, then the theoretical difference we have identified between forwards and futures would not be present. In terms of Table 1, there would be no liquidity difference between standard forwards and standard futures, so we should expect the liquidity premium to be priced elsewhere. This could be in instruments that do not appear in Table 1, such as currency swaps, or in *ad hoc* arrangements made directly between banks and their clients. That we do find the liquidity-risk premium to have explanatory power for interest rates suggests that liquidity risk has indeed been priced into the spot interest rates from which we derive our implied forward rates.

7 Conclusion

We have sought in this paper to understand a fundamental payments problem, namely the mismatch between banking clients’ natural forward positions arising from their business in the real economy. Banks act to address this mismatch by offsetting different clients’ cash flows against one another. When this is impossible, banks can go to the forward and futures markets to try to achieve more complete offset there. The liquidity and depth of these markets comes at the cost of decreased
flexibility in terms of contract dates, maturities and face values, so banks may still find it impossible to offset all cash flows. Since different means of offset differ in their detailed cash-flow characteristics, we can interpret their price differences as reflecting the price of the underlying liquidity risk which they help to offset.

Our work could be extended by interpolation of the interest-rate or futures-price series that enter in our analysis in order to obtain a more continuous data set, as is done by Grinblatt and Jegadeesh (1990). This would allow comparison of futures and forward prices on many more days than is possible without interpolation.

Another avenue for future research is suggested by the observation made at the end of Section 6.5—an extension to other interest-rate and exchange-rate instruments. The analysis undertaken here is a measurement of the price of liquidity risk deriving from instruments having different cash flows. The interest-rate and foreign-exchange markets have numerous instruments other than those we have studied here, each with its own specification of cash flows. In particular, interest-rate and currency swaps come to mind, and informal evidence suggests that these have come to hold much of the liquidity in these markets in recent years. A logical continuation of the analysis undertaken here, then, is to fully understand the institutional arrangements of those instruments (especially along the lines of Tables 2 and 3) and to measure the price of liquidity risk there. This could be viewed as adding columns to our Table 1 and work in this direction is undertaken by Johannes and Sundaresan (2007). With private data, one could also move left in those tables, to understand the price banks charge for the internal bearing of risk. Relation of these further measures of liquidity risk to macroeconomic problems would support our larger research aim of understanding the role of the banking system in monetary and macroeconomics.

References


